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CHANGES IN REAL ESTATE VALUE DYNAMICS DURING SERVICE CYCLES AS A SOURCE OF NEGATIVE PERIODIC PROPERTY DEPRECIATION

Abstract

Main methodological principles of mathematically describing the patterns of changes in the asset's value/depreciation dynamics are studied in cases when economic measurements are performed by independent expert evaluation. The basic hypothesis suggests that for all tangible assets, which are characterized by redeemable depreciation, there is a possibility of negative periodic depreciation during short-term service periods when remedial and repair work to eliminate depreciation signs is carried out.

The most influential price-forming factors that determine the asset's depreciation indexes and indicators of value dynamics over long periods are identified and analysed. It is shown that when this period is comparable to the asset's service life, most of tangible assets are characterized by both positive and negative

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periodic depreciation indexes at separate times. It is noted that the models used in accounting documents do not describe the actual changes in the value dynamics, and amortization in particular, since they do not take into account the possibility of increasing asset value and periodic negative depreciation. A new kind of mathematical model is proposed that takes into account the opposite signs of periodic depreciation in the operation and service periods. It is proved that the actual indicators of fair market value and periodic depreciation indexes of these types of assets can be determined by performing periodic independent expert evaluation (revaluation).

Key words:

independent expert evaluation; market value; accounting; service life of an asset; operation and maintenance stages of asset life cycle; obsolescence; depreciation; Iowa curves.

JEL: C13; C49; E30; E37; O34; O47.

Introduction

The methods of mathematical simulation/modeling are widely used in econometrics – in particular in its practical applications, when economic measurements of asset value are performed by independent expert evaluation. This creates conditions for determining the fair market value of assets by indirect methods and using available market source data, when its experimental determination by the direct method is not possible (Magnus et al., 2004, p. 475). On the basis of such indirect measurements, it is possible to monitor the value of any specific real estate assets and to determine any patterns in the changes of asset value in the long term. Conversely, when using a costs approach in performing valuation, the depreciation should be determined in accordance with these patterns. Therefore, an in-depth study of dynamic changes in real estate value is appropriate from both a theoretical and practical point of view.

Problem statement

Mathematical models currently used in the independent evaluation that determine the dynamic changes of asset value in the long term generally consider only long periods of asset operation, excluding short-term maintenance periods, when repair and remedial actions are taken. Generally accepted simplified models do not take into account periods of any repairs, reconstructions, routine maintenance works – when the valuation object cost increases. However, such works performed during those short service periods significantly change the overall trend of changes of value over time, as well as the quantitative characteristics of asset value and depreciation. Generally, it is assumed that dynamic changes in real estate value tend to result in monotonic value decrease, reflecting the trend of asset obsolescence, which is caused by the value loss proportional to the increase (constant as per traditional models) in depreciation. From our point of view, a qualitatively different process takes place during the maintenance periods – namely, the asset value is abruptly and sharply increased, as a result of the works performed to eliminate the manifestations of depreciation.

Accordingly, depreciation is partially reversed in such short periods for assets, in which manifestations of wear can be repaired or restored. In fact, long periods of assets normal operation with positive depreciation, which are exclusively considered in the professional literature in the analysis of depreciation indicators, alternate with short maintenance periods, when these indicators take negative values. Moreover, these maintenance periods, with their short duration, compared to long-term operation periods, until now have not been adequately reflected in the models used to describe estimated change in asset value over time. However, the impact of increased asset value caused by such periods is extremely significant and cannot be ignored in the depreciation dynamics analysis. Neglecting this important feature of the asset value dynamics function leads to an inadequate interpretation of the patterns in the change of market value over time – and, consequently, to misrepresentations of the current value index at the valuation date. This reduces the accuracy of the evaluation due to the increased uncertainty of the evaluation result. According to Section 4.3 clause 4.3.6 of *The Valuation Report* of the European Valuation Standards (2016, p. 166), «Where the market for the property being valued is affected by uncertainty and this is relevant to the valuation, the valuer should comment on the reasons and degree of uncertainty within the report».

Therefore, it is necessary to conduct an in-depth study of changes in property value and depreciation dynamics in order to present a theoretical justification and analysis of a more accurate model of their description, taking into account the actual presence of negative depreciation indicators occurring during the maintenance periods. The urgency of this problem is undeniable, since reducing the uncertainty of evaluation results is one of the main priorities of economic valuations.

Literature review

A number scientific works have given considerable attention to research on depreciation and patterns of changes in asset value over time. N.V. Mirzoyan (2005) points out that the appraisal concept of depreciation in independent expert evaluation means the loss of asset utility, and therefore – its market value decrease. Author identifies two possible ways of calculating the depreciation index: the service life concept and the depreciation breakdown method. The latter is used for determining the probable service life of property as the sum of asset's current age index and the probable life expectancy at this age. The calculation of depreciation index on the basis of service life concept examines the item being valued under the assumption that its effective age relates to typical service life the same as the accumulated depreciation relates to its replacement value. Aggregate accumulated depreciation is considered as a function of asset's service life.

The most common method is the breakdown into three types of depreciation, in which the sum of all possible depreciation types is considered as an indicator of the property accumulated depreciation (Mirzoyan, 2005, p. 111). Two methods are used to determine the accumulated depreciation factor: additive and multiplicative. In the additive approach, the value of the accumulated depreciation total coefficient is determined by summation of three types of depreciation coefficients – physical depreciation/deterioration, functional or moral obsolescence and economic or external obsolescence. It is noted that this method can only be used in limited conditions, when the values of the three types of depreciation coefficients are small. Otherwise it results in an indicator of total accumulated depreciation greater than 100% – which is considered by the authors (Mirzoyan et al., 2017, p. 654), as contrary to the market value definition and completely contradicts to the physical aspect. Therefore, due to the equal significance of the three types of depreciation coefficients in the formula of the additive method (Mirzoyan et al., 2017, p. 654) and its simple arithmetic summation, this method does not correctly reflect the form of each particular depreciation type's impact on the asset's value. The multiplicative approach is based on the formula (Mirzoyan et al., 2017, p. 654), which with the same variables gives a completely different result of accumulated depreciation coefficient. P. V. Kartsev (2001) and P. Ya. Balakin (n.d.) use a similar formula for the multiplicative approach to determining the total accumulated depreciation index by distinguishing the three above-mentioned types. A. O. Alekseev (2011) gives an overview of accumulated depreciation calculating methods, taking into account the change of asset's qualitative characteristics. He states that the combination of these methods will allow scientists to, firstly, determine whether the accumulated depreciation is increasing or not, secondly, take into account the requirements of potential buyers and their ideas about the factors and processes of asset depreciation, and thirdly, take into account actual market conditions and its trend dynamics.

Depreciation is also closely related to bookkeeping amortization, used in accounting, but these concepts are not identical. Amortization is a legal set of statutory accounting transactions rules, performed according to the established formal algorithms, as per which depreciated property value gradually decreases in accounting reports. In this way, the amount of additional capital is determined, intended for enterprise updating and replacing worn-out property assets. On the other hand, depreciation is a completely objective value change, which does not depend on the adopted accounting rules. Therefore, accounting amortization is only a secondary formal procedure that must be based on objective depreciation indexes. Ideally, amortization should reflect actual changes in asset value dynamics – but due to the incomplete accuracy of the generally accepted amortization models with regards to the real function of depreciation dynamics, some divergence between them arises. During useful life of the valued item, the amortization dynamics may be very different from the actual depreciation and market value. The most striking example of such a deviation is accelerated depreciation model, when full property amortization does not mean that it is completely worn out.

Accelerated depreciation model uses the «declining balance» method of calculating depreciation, which assigns more of the depreciation expense to the earlier years of the asset's life. In the accelerated amortization model, the effect of accelerated non-linear depreciation is achieved by applying a higher than linear amortization rate and applying it to the enterprise's balance. This means that the prior years' accruals are deducted each year to yield a declining balance. Where estimates of asset service life are subject to uncertainty with large margins of error, the declining balance method has an advantage because a relatively smaller amount of amortization is left in the remaining period of asset's service life. In addition, this method generates more internal funds from depreciation accruals, as long as total overall enterprise balance value continues to grow.

In some cases only, amortization rates can be used in the calculation of depreciation rates, since sometimes they are approximately the same and usually amortization data are easily accessible and documented in accounting reports (Kovalev et al., 2006, p. 38). On the other hand, amortization rates are in complete conflict with real depreciation indexes for certain types of intangible assets. According to the accounting documents, the amortization accrues are calculated on the value of such an asset, which results in its accounting value being reduced. At the same time, its market value is actually growing, demonstrating negative depreciation. There is no legal possibility for negative amortization, caused by this negative depreciation, to be reflected in enterprise accounting reports (Pozdnyakov & Lapishko, 2018, p. 728; 2019a, p. 393). Such a glaring contradiction can only be corrected by performing periodic assets revaluation, and thus bringing asset value in the accounting records closer to the real fair market value that corresponds to the actual function of the asset's depreciation dynamics. However, the latter can be reliably determined only by performing economic measurements with independent valuation methods.

The division between redeemable and irredeemable depreciation is considered by N.V. Veig, who establishes that it depends on the technical ability and economic feasibility of eliminating manifestations and signs of depreciation. Therefore, intrinsically there are two types of depreciation: redeemable and irredeemable. Irredeemable depreciation occurs when repair costs (repair of buildings, replacement of equipment parts and components) are higher than the property value and usefulness increment. The cause for irredeemableness of depreciation may be the object design, constructive, space-planning, technological features or its maintainability. Redeemable depreciation is the type of depreciation for which repair costs are lower than the costs attached to market value of the asset. Repair, restoration and renovation are ways of eliminating physical depreciation, while reconstructing, restyling, updating and upgrading asset characteristics and specifications are ways of eliminating functional devaluation (Veig, 2009, p. 49). S. V. Gribovsky (2001) emphasizes that from the market point of view, the best way to assess asset depreciation is to analyse its sale price dynamics throughout its entire service life. The complexity of conducting such research for real estate is caused by a sufficiently long service life of the investigated assets, sometimes equal to the duration of human life.

An alternative way of conducting such research may be identification of the dynamic changes in the present value of the future income flow that this asset is capable to generate (Gribovskiy, 2001, p. 64). The nonlinear model of value change over time is typical for buildings and constructions, characterized by a slow decline in value at the beginning of the service life and accelerated decline at the end. It is described by a convex upward curve and is defined as progressive depreciation, the rate of which increases over time. From an economic point of view, such a change in the asset value can be explained by the fact that the initially created/acquired property item is in line with modern market requirements at the creation date, has high and fairly stable market demand for some time due to its novelty and high consumer utility. As a result, its market value decreases slowly at the beginning of its service life. Moving closer to the end of the service life of the asset, due to the gradual loss of usefulness and accumulation of defects, more and more manifestations and signs of its functional, economic and physical depreciation appear, which leads to an accelerated depreciation rate and, consequently, to even more significant value change. As time passes, the discrepancy between the actual and the most effective use of that asset will increase, which may be considerable due to external (economic) depreciation factors (Friedman & Ordway, 1981). Market requirements and technological standards in building construction change much faster than physical and consumer qualities of property. After a while (for profitable real estate, in general, no sooner than 40 – 50 years), the profitable asset begins to lose its usefulness in general and investment attractiveness, becoming inferior in terms of consumer characteristics to more modern analogues and, accordingly, its market value begins to decline rapidly. Therefore, progressive depreciation can be defined as loss of asset value at an increasing rate. Thus, the curve of change in the value of real estate under this model of depreciation is characterized by a slow fall first, but subsequently, after a certain period of time, an increasingly rapid drop. That is why this type of depreciation qualifies as progressive, with functional depreciation being dominant in integral accumulated depreciation index. There are three main models of depreciation in general: the first is progressive depreciation, the second is conditionally linear depreciation that corresponds to the linear amortization model in accounting, and the third is regressive depreciation (Gribovskiy, 2001, p. 72).

Regressive depreciation is a completely different kind of value change over time, typical for machinery and equipment. The regressive depreciation model is characterized by a rapid value decline at the beginning of the service life and a slower one at the end of it, and it is described by a concave upward curve in the time-value coordinates (Kozlov, n.d.). The author notes that the method does not take into account operation/maintenance cycles and individual events that occurred during the service life of the valuation object. Admittedly, this restriction applies to all other studies, although it is clearly formulated only in this one case – obviously, for the reason that the author is aware of the importance of analysing the changes in cost/depreciation dynamics during the aforementioned cycles. Other works consider only smooth curves of monotonic functions in separate periods of operation. They do not take into account the break points of these monotonic functions at occurrence of individual events such as repairs, reconstructions and renovations of real estate, emergency and intentional operation breaks of machines, repair and modernization of technological equipment, etc. From our point of view, proper depreciation dynamics consideration of such events and periods would make it possible to improve models of theoretical description of patterns in changes of asset value over time and make it more realistic. This seems urgently needed, because during these short periods there is a sharp increase in asset value, demonstrating negative depreciation indexes and creating conditions for extended asset life. Mathematical models of value changes over time have been considered most fully in works of Robley Winfrey (1935; 1942a) and his followers (Marston et al., 1953; Henderson, 1968; Trishin, 2005).

The aim of the study is to test the working hypothesis that all tangible assets that are subject to redeemable depreciation have occurrences of periodic reverse depreciation – but only during short periods when remedial actions are taken. Consideration of the actual patterns of changes in real estate value over time, taking into account operation/maintenance cycles on a specific example of current valuation practice. Development of a more accurate model of property value and depreciation change functions, describing the general pattern of both the asset value loss resulting from periodic increase in its depreciation, and of its value increase due to periodic decrease in depreciation indicators in periods when its manifestations and signs are eliminated. Theoretical justification for the periodic occurrence of negative depreciation in the maintenance periods of repair, reconstruction, modernization of real estate objects, when their value during short-term periods increases.

Research results and discussion

As we can see from the literature review, much attention is paid to the models of accumulated depreciation of assets and analysis of value changes over time. However, the studies mainly consider the accumulated depreciation indicator during the time intervals in which the valuation object is used/operated and are not analysing periods when any other activities in the form of repairs, reconstructions, upgrades, aimed at eliminating depreciation signs and increasing the object value, are carried out. It is clear that such individual events being excluded from the consideration significantly simplifies the picture describing the patterns of changes in asset value over time. This simplification limits the analysis to simple monotonic functions without any breakpoints that correspond to these events. As a result, all information sources analyse only simplified models of changes in asset value over time, which do not take into account periods of their value increasing.

It is our deep conviction that refusing to consider such events and periods substantially distorts the real picture of changes in asset value/depreciation dynamics. In fact, during those excluded periods, there is a sharp and abrupt increase of asset value that completely changes the nature of the studied time dependencies. Negative depreciation indicators are especially observed during those periods of eliminating depreciation signs, and those same short intervals are the periods of asset value increase. They are relatively short when compared to entire service life of the asset, but they are very important for its technical and economic indicators, including utility, productivity, value, depreciation, expected service life and so on. Therefore, the smooth curves of long operation periods, excluding maintenance cycles, give a too simplistic picture of asset value/depreciation changes that has little to do with actual performance over the longer term of the asset's service life. On this basis, it is suggested that when analysing and determining such indicators, it is imperative to take into account the curves' break points of that model's monotonic functions. Essentially, we are proposing to use a new class of mathematical models of changes in asset value over time, which would take into account the short periods of negative depreciation arising from the occurrence of repairs, reconstructions and renovations of real estate, emergency and planned stops of machines and mechanisms for adjustment and routine preventive maintenance, repair and modernization of technological equipment, etc. Taking into account the sharp changes in the asset value/depreciation during these periods of negative depreciation allows us to develop more realistic models of theoretical description of change patterns in the dynamics of studied parameters.

The basic principles of independent expert evaluation and the theory of fair market value formation are the theoretical background of the proposed approach. From the point of view of investment analysis fundamentals, the aforementioned measures result in increased capitalization of valued asset, which is possible through additional investments in the asset aimed at eliminating the depreciation signs and making inalienable inherent improvements.

The widespread simplified models of changes in asset value over time that do not take into account periods of value increase are perfectly adequate for the limited operation periods, when no events related to the elimination of depreciation signs occur. However, such models cannot be used for long-term periods equal to the entire service life of the asset. Neglecting the objective presence of negative depreciation at separate short intervals introduces considerable error in the analysis results, forcing analysts and evaluators to use continuous smooth curve functions without any breakpoints in situations, where they are actually broken during periods of negative depreciation. For both real estate and machinery/equipment objects, such individual events are a fully accepted periodic regularly performed occurrence, that is an integral part of the usual life cycle.

As valuation practice shows, repairs, reconstructions, and upgrades are usually carried out even more often than required by economically motivated circumstances. As a rule, in almost every real estate transaction, the new owner immediately after obtaining the property rights seeks to invest some funds (to a smaller or bigger degree) in improving the property's functional characteristics, adapting the acquired property to new needs, meeting more modern market requirements and realizing his own ideas regarding possibilities of future use. In addition, the need to invest money securely, the rational desire to profit maximization, the drive for business expansion and desire to be more responsive to the changing market environment conditions often force the owner to periodically make some additional improvements to the commercial property. Refusal to consider the impact of these individual events on the model of changes in asset value over time has led to the fact that the indisputable presence of negative depreciation caused by value increase in certain short-term periods has been ignored. The inconsistency of traditional simplified models with the reality, due to their failure to take into account the periods of asset value increase, limits their use to only those periods of time when no work of eliminating depreciation signs – like repairs, reconstructions and upgrades – is done. Accordingly, the use of such models over the long term causes quite obvious discrepancies between actual and model value/depreciation indicators, which significantly hinders the accuracy of the valuation performed based on these models.

Let's consider the features of the proposed approach in more detail by comparing it with traditional models of describing changes in real estate value and depreciation dynamics. Robley Winfrey developed the concept of determining present property value based on the assumption that its value at any age, at any chosen valuation date, is determined by the value of the property at zero age and the present worth at that date of operating returns on earlier made investments amount, that will be received within the property's residual expected life. The amount of the operating returns on earlier made investments for the appraised property cannot be determined directly, as it is only an uncertain part of the larger consolidated income flow produced by the property. Instead, the initial value of property at zero age in a very obvious way depends on the costs of acquiring/creating new property (Winfrey, 1935; 1942a). These prerequisites have become the basis for the methodology of determining the terms of service life based on Iowa curves/Survivor curves (Marston et al., 1953; Trishin, 2005). They were developed by statisticians from the Iowa State University in the United States as a result of a study of empirical data on property retirement, determining the number of property objects that have remained operational over the average expected life for machinery and equipment, real estate, engineering networks utilities infrastructure, etc. The purpose of these studies was to develop some practical models for reasonable calculation of service life of machinery and equipment, based on the statistics of gradual retirement of operated objects when they reach a certain age. The essence of the method was the analysis of useful life data of the representative property group of same class, which began to operate at the same time and were operating in approximately the same conditions. Next step was an analysis of long-term statistics about how many objects in this group became worn out each year and were retired. The percentage of objects that ended their service life each year was calculated from the initial number of objects in that group. According to the obtained data, first an empirical graph of the stub function was constructed, which was further approximated by the smoothed curve (Nechiporenko et al., 2016, p. 102).

The basic concept of present property value states that its value at any age is estimated, in particular, by the present value of the investment's operating returns over its future residual useful life, the amount and duration respectively of which is not known to any degree of certainty. To overcome this uncertainty, the principle of financial equivalence was formulated, which states that the financial equivalent of future operating returns over the expected residual life of the property can be calculated on the assumption that all the various options for future operating return curves is estimated as the cost of acquiring/creating new property. Robley Winfrey stated that the most convenient, though not necessarily feasible, was the assumption that the sums of a homogeneous annuity operating returns series are equal in the amount to the variable annual future operating returns over the entire expected life of the property. The assumption is that at continuous use property users are obliged to pay annual shares of amortization/depreciation deductions from net operating returns for such a long time that after the end of property's life the amount of deductions will be sufficient for its replacement with a similar new property of the same quantity and quality. Given that future operating returns may vary in size, the author emphasized that the levels of future annual operating returns may be different, but they could all be represented by an equivalent indicator of homogeneous annual operating returns with such a spread that the sum of the present value of all operating returns over the whole service life would be equal to the sum of the previous/current values of annual operating returns over the same life period (Winfrey, 1942a). Using the above concept of equivalence, for each age of the asset, the author proposed a factor called the «condition percent factor», by which the amount of acquiring/creating new property value, or the cost of new property, can be converted to the present property value index at the present age. Robley Winfrey (1942a) defines the concept of «condition percent factor» as «the ratio of the present value of the depreciable property relative to its depreciable value when new». Tables giving the numerical values of the condition percent factor for some kinds of properties, at different ages for different probable service lives and annual discount rate of net operating returns, have been published in (Winfrey, 1942b). In a later publication with co-authors, the condition percent factor was called «expectancy-life factor».

The analytical expression for this factor obtained by Robley Winfrey has the following form:

$$c = \frac{(1+r)^n - (1+r)^x}{(1+r)^n - 1}, \tag{1}$$

where c – condition percent factor,
n – probable asset service life in years,
x – current age of the asset in years,
r – annual rate of net operating returns.

The author defines condition percent factor c as the ratio between the property present value V_i at the current age x, and its initial value V_0 of the new property:

$$c = \frac{V_i}{V_0}. \tag{2}$$

From (2) we can easily obtain a formula for calculating the present property value V_i at the valuation date, at its current age x:

$$V_i = V_0 \times c. \tag{3}$$

Thus, the mathematical definition of the model of change in asset value over time is full and definitive, and expressions (1) and (3) are necessary and sufficient conditions for it. Based on this model, it is easy to determine the characteristics of accumulated depreciation, which we will hereafter refer to as total absolute depreciation $A_{tad\,i}$:

$$А_{\text{tad i}} = V_0 - V_i \,, \tag{4}$$

where V_i – estimated asset value index in year *x*, at the valuation date;

V_0 – initial asset value indicator in year zero, at the date of asset's creation or inclusion to the balance sheet accounting report,

and the accumulated depreciation factor $К_a$:

$$К_a = 1 - \frac{V_i}{V_0}. \tag{5}$$

By substituting (3) into (4), after elementary transformations, we obtain

$$А_{\text{tad i}} = V_0(1 - c). \tag{6}$$

Accordingly, by using the expressions (3), (5), we arrive at

$$К_a = 1 - c. \tag{7}$$

For the sake of analysis completeness, we also consider the absolute annual depreciation index (or, shorter, the annual absolute depreciation) $А_{\text{aad i}}$, which for the *i*-th year is defined as absolute annual increment of the asset value taken with the reverse sign and measured in monetary units:

$$А_{\text{aad i}} = V_{i-1} - V_i \,, \tag{8}$$

where V_i – estimated asset value index in year *x*, at the valuation date;

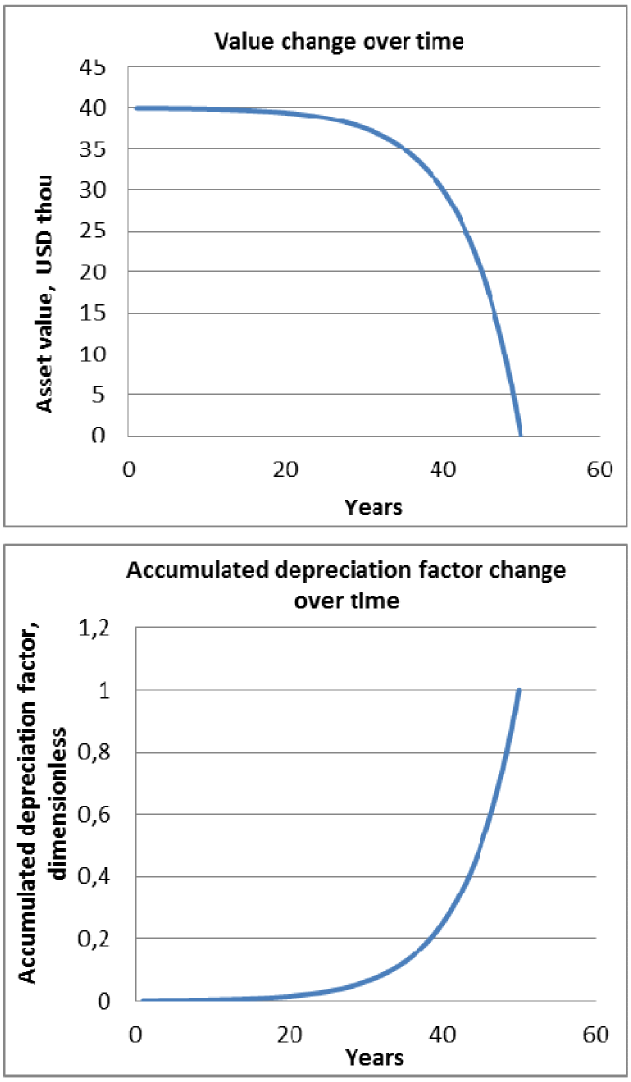
V_{i-1} – estimated asset value index in the previous *(i – 1)* year.

In a wider context, it would be more appropriate to call annual absolute depreciation $А_{\text{aad i}}$ «periodic absolute depreciation», since the year is only one of the possible variants of duration. The methodology for calculating the depreciation indicators used above and their mutual relationship are discussed in detail in (Pozdnyakov, 2019, p. 91).

Below a graphical interpretation of the defined above (6), (7), (8) depreciation indicators $А_{\text{tad i}}$, $К_a$, $А_{\text{aad i}}$ is presented, on an example close to valuation practice – industrial property with characteristics V_0 = $ 40,000, *r* = 0.15. The probable term *n* of expected service life for the capital group IV buildings is 50 years.

Figure 1

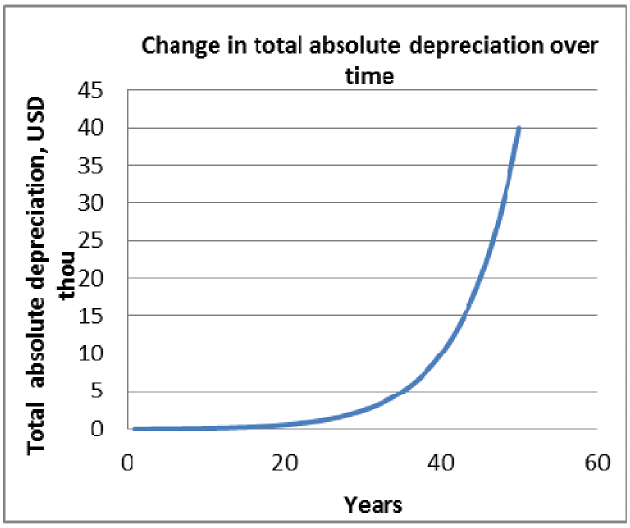
Change in property value over time (left) and change in the property accumulated depreciation factor over time (right). Traditional model

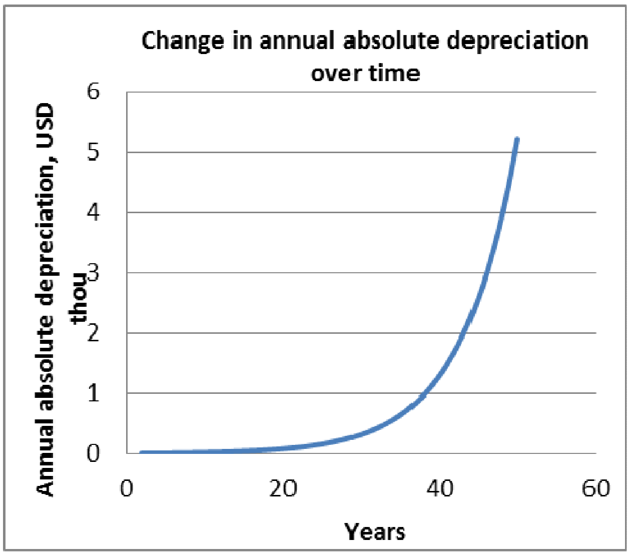


Source: designed by the authors.

Figure 2

Change in the property absolute accumulated depreciation over time (left) and change in the annual absolute depreciation over time (right). Traditional model





Source: designed by the authors.

As we can see from fig. 1 and fig. 2, when using a well-known traditional model of asset value change over time, we get dependencies described by smooth continuous curves that do not take into account periods of depreciation elimination. The pattern of changes in property value over time is described by a sharply nonlinear convex upward graph with negative first and positive second derivatives, and it corresponds to the definition of progressive depreciation.

The traditional model also does not consider the operation costs and their effect on the date of asset's retirement. According to the known model, at the end of asset's service life its residual value asymptotically approaches to zero. In fact, the asset is retired much earlier, for example for equipment this is at a certain non-zero scrap value. However, the discussed above traditional model provides for complete exhaustion of the valued asset only when zero value is reached, and this date is considered to be the end of asset service life. It is clear that such a description is idealized and very simplistic. Its accuracy to reality is observed only in isolated fragmentary periods not close to the end of asset service life, and when repairs and upgrades are not carried out. Therefore, this model does not correspond to reality for a long period, such as the asset's entire service life.

To eliminate the above-mentioned disadvantages of the traditional model and to obtain a more adequate description of the analysed value/depreciation indicators, in accordance with the theoretical justification above, we propose to complement the evaluation above with the following. Let us suppose that during the 40^{th} and 45^{th} periods of the asset's existence, repair and restoration works were carried out to eliminate depreciation manifestations, amounting to \$5,000 and \$10,000 respectively. In full accordance with the basic valuation principles of *contribution* (*marginal productivity*) and *change*, the value of the asset will increase, at first approximation and in normal conditions, at least in the amount equal to the investment costs. The principle of *contribution* (*marginal productivity*) establishes that the property value is increased through introduction of any additional factors that may increase the asset's value from the consumer's point of view. Marginal productivity is not identical to the actual investment costs of additional, inherent improvements, as some additional factors may increase the property value by a much greater amount than the costs associated with property improvements or consumer characteristics changes. One example would be an inclusion of a garage block or swimming pool into a residential complex, if the cost of constructing them were lower than the resulting total increase of the entire residential complex's value. The principle of *change* states that the market value of an asset is not constant and may change over time. A variety of factors influences real estate usefulness and, accordingly, its value from the perspective of a potential consumer. First of all, it is the degree of their comfort, usefulness and general compliance with market requirements, as well as the asset's depreciation index and expected time of its ability to be used effectively. Changes in external economic and market conditions over the relatively long period of real estate use are also of great importance. Changes in production technologies, social priorities, demographic situation, settlement status, pervasive stereotypes in the public consciousness, international production distribution, national macroeconomic policy, and many other factors of the multifactor market space can significantly affect the degree of usefulness, utility and market value (Friedman & Ordway, 1981).

The degree of increase in the valued asset's price depends on the effectiveness of investment funds use. In practice, the remedial works to eliminate depreciation signs can cause in the asset's market value to increase by either greater or less amount than the repair costs or money invested. In the second case, some works can be performed that qualify as redundant inherent improvements that is economically unreasonable changes in terms of the evaluation theory. Similar situations occur when an owner, who does not have complete information about current market requirements, makes decisions at their own discretion, without contacting experts in real estate valuation or investments consultants. In this case, some of the investment may be misused, and unnecessary works may be performed that do not add anything to the total value of the asset, or increase it only slightly.

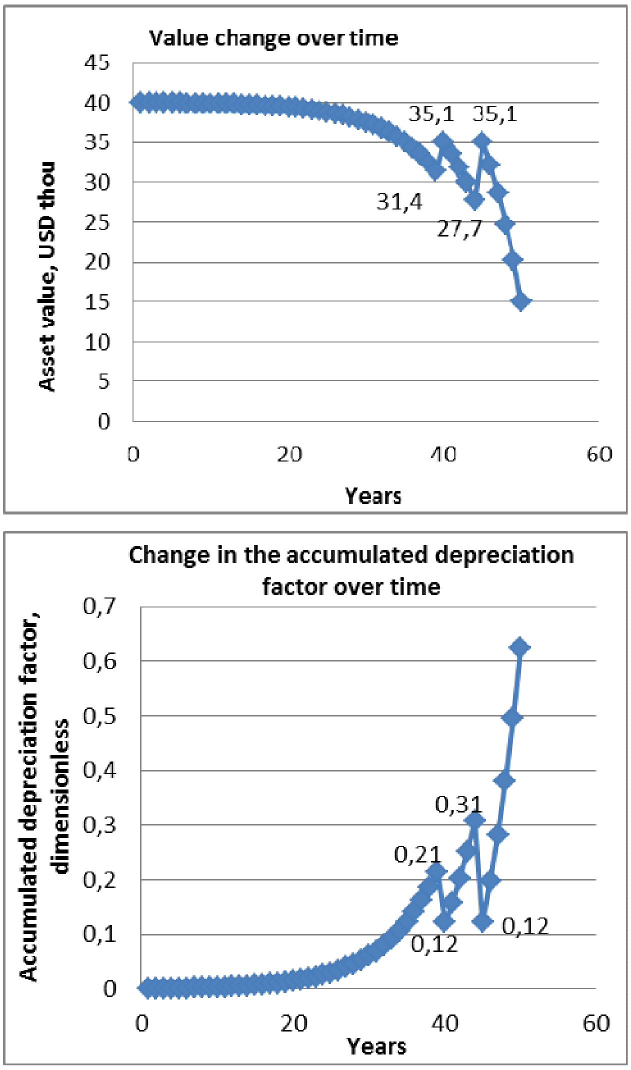
In the example considered here, we analyse an intermediate case – we provisionally consider that the repair costs are taken into account with a single factor and they are algebraically summed up with the annual depreciation cost. As mentioned above, the investment funds contribution factor can in fact be both greater and less than one, depending on the degree of the performed work's compliance with the actual market requirements at the date of their implementation. Below the calculation results under the above-mentioned conditions are shown in the form of graphs.

As can be seen from Fig. 3, 4, by taking into account the real conditions of the asset's operation, the previously shown dependences with smooth continuous curves are transformed into more complex functions with a few breakpoints, which reflect the effect of abrupt changes in indicators. At the same time, during the period of repair works, the asset's indicators of annual absolute depreciation become negative, which is quite understandable, since the asset's value does not decrease during these periods but, on the contrary, increases. In the described above way, we move from a simplified idealized mathematical description to the more accurate model that is a real reflection of analysed indicator dynamics over the long term.

The graphs shown above are based on a number of assumptions that must be specified. Firstly, it is assumed that the duration of the depreciation repair works is disproportionately short, in comparison with the asset's service life. Therefore, the annual absolute depreciation $A_{aad\ i}$ has a negative sign only at the curve point that corresponds to the moment of implementation of these works.

Figure 3

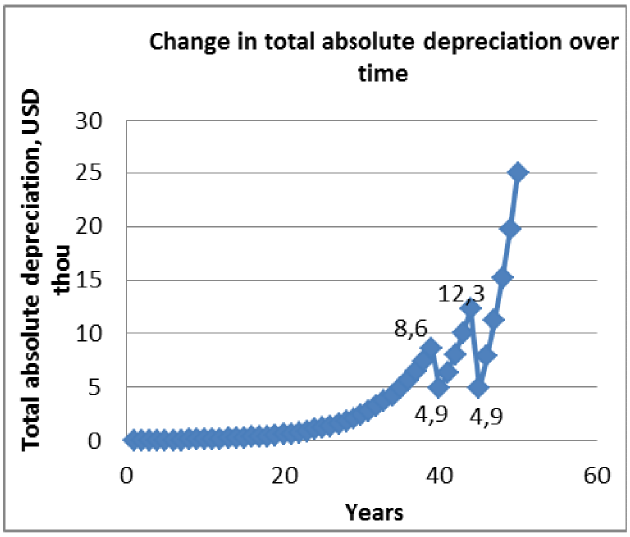
Change in property value over time (left) and change in the property accumulated depreciation factor over time (right). Proposed model

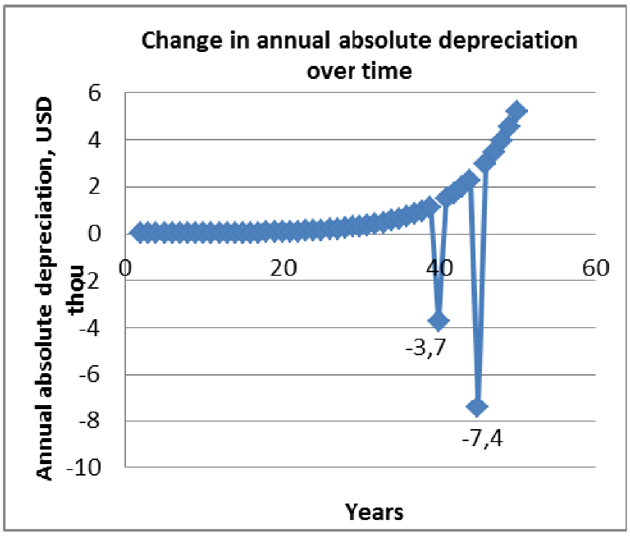


Source: designed by the authors.

Figure 4

Change in the property absolute accumulated depreciation over time (left) and change in the annual absolute depreciation over time (right). Proposed model





Source: designed by the authors.

This means that in a year, for example, depreciation removal work was carried out during one month, for which time opposing calculations will both be taken into account. On the one hand, the accumulated depreciation increases as per model expressions (1), (3), because of the asset's value decrease. On the other hand, this depreciation decreases because of the implementation of the aforementioned works. Thus, the obtained indexes of present asset value, total and annual absolute depreciation are determined by the cumulative effect of both factors with opposite signs and directions of value change. If the duration of depreciation removal works is longer and lasts over one or more periods, the annual absolute depreciation is negative throughout such time, provided that the schedule of costs payment for the performance of these works is linear.

Secondly, we believe that the pattern of changes in property value over time after the implementation of depreciation removal works remains the same as determined earlier for that property, and that this pattern will be continue even after the works are completed. In the case of a high nonlinearity degree dependencies – such as those analysed in the example above – this is essential because the kurtosis of characteristics is significantly different at the beginning and as the end of the asset's service life.

The above assumptions are valid for the relatively small scope/costs of works – but for the larger and more costly works, these assumptions will be less grounded. In the case of the large scope of works, concerning main structural elements of a building or engineering networks, it seems more appropriate to begin a new count from zero accumulated depreciation at the date of the reconstruction completion. If the large-scale works are done well enough to be high quality, the actual pattern of further changes in value over time will be more similar to the conditions of a newly created object – first a long period of slow increase in accumulated depreciation, followed by an accelerated increase only after a sufficiently long period of operation has passed since reconstruction.

Conclusions

The paper proposes and examines a new class of mathematical models to describe the patterns of asset depreciation and value changes over time, taking into account the negative depreciation occurring during short periods of performance of depreciation eliminating works. It is shown that during asset repairs, reconstruction, routine preventive maintenance and technological equipment modernization, the value of these assets increases and changes in depreciation dynamics occur, as in the indicators are opposite in sign to those of operation periods. Traditionally, the models used until today did not take into account said changes in asset value/depreciation dynamics and did not consider periods of negative depreciation. The proposed improvement to the mathematical model describing the patterns of the asset's value/depreciation dynamics made it possible to provide a more accurate mathematical description of the regularities in the studied parameters' changes over a long time equivalent to the duration of asset's service life.

The main result of this study is evidenced confirmation of possibility to increase the accuracy of the mathematical description of patterns in changes of asset value and depreciation over time, which is extremely important for the purposes of determining the correct current values of these indicators at the valuation date. The performed theoretical analysis has shown that during the service life of real estate property, machines and equipment, their market value may change in both directions. In general, over the long term, it changes mainly downward, showing periodical positive depreciation and overall loss in asset value. However, during short-term periods, when works on eliminating depreciation are performed, their cost changes promptly in the direction of increase, showing negative depreciation. The evidence given in the article provides grounds for assertion that the real value-time dependence model for the abovementioned assets is much more sophisticated and complicated than previously thought, and it must contain alternating periods of positive and negative depreciation in the long term.

Models of changes in value over time for these classes of assets that are established by accounting documents and used for calculation of amortization obviously do not correspond to the factual state of affairs and do not take into account the possibility of increase in property value during some periods. The discrepancy between bookkeeping values of assets reflected in accounting reports and their real fair market value may be eliminated by periodic revaluation of assets, using the mechanism and techniques of independent expert evaluation. This conclusion is in full agreement with the opinion of Alfred M. King (2006), who claimed that some intangible assets, in particular trademarks and brands, have indefinite service life. He believed that the amortization on such intangible assets seems to be unsubstantiated: after all, as long as the company owner is promoting its brand, its value is increasing constantly. Companies cannot reflect the increasing brand value in accounting and financial reports, but the least they could do is to not reduce its value through amortization as long as there are future prospects of its continued effective use for indefinite time. As we can see from the above analysis, the alternating sign-changed depreciation over long-term period with a certain extent may be applied to tangible assets other than property too. The advisability of performing periodical revaluation of such assets to approximate accounting data closer to fair market value index is also confirmed by other studies (Fishman et al., 2002). The use of the aforementioned approach to reduce the uncertainty of valuation results creates the opportunity business leaders to use it for optimizing management, accelerating the pace of development and long-term planning of enterprise strategies (Omae, 1983).

In order to reduce the uncertainty of valuation results, a more accurate mathematical model with break points of value/depreciation dynamics monotonic functions has been proposed and tested. It is proved that in the most general case, over a long term, against the background of consistently positive accumulation of periodic depreciation, there is a possibility of temporary short-term negative depreciation caused by increases in asset value during some periods when depreciation signs are eliminated. In general, this does not change the overall downward trend of the asset value, but it has a significant impact on the quantitative indicators of the value/depreciation dynamics. Taking into account these features, the valuation uncertainty is reduced and determination of value indicators becomes more accurate in cases when repair works are performed. The results obtained in the study may be useful in developing and validating practical methodology of asset valuation. The reliability of valuation results directly depends on the accuracy of the models used and, in particular, on the appropriateness of the chosen model of changes in the asset value/depreciation over time.

Quantitative analysis of positive and negative periodic depreciation indicators as equivalent components of total accumulated depreciation of assets is of interest for further practical and theoretical research in this area. Studies on calculation methods for asset's estimated residual life are of particular interest, as it depends directly on the above components and characteristics. We consistently adhere to the view that measurements of all parameters, including economic ones, use the same laws for the selection, transformation and transmission of measurement information (Pozdnyakov & Lapishko, 2019b, p. 80). The completed research is another next step in direction of developing an information and metrological paradigm of independent expert evaluation, which, in our opinion, is the most promising way of further improving this methodological framework.

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