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SECTIONING OF HIGH DIMENSIONAL BANDED MATRICES

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Abstract: *Solving high dimensional systems of linear algebraic equations is of use to many problems of mathematical physics, in particular, it is one of the main subgoals at solving systems of equations in partial derivatives. Distributed solving of high dimensional systems of linear equations allows to reduce computing time, especially in cases when these matrices can not be kept in one computer's RAM. The subject of this study is the search of optimal high dimensional matrices sectioning algorithms for distributed solving systems of linear algebraic equations.*

Keywords: *Sectioning, SLE, Distributed Computing, Thermal Design.*

1. INTRODUCTION

Many problems of mathematical physics can be described by a system of equations in partial derivatives. After linearizing such systems with numerical methods, problems come to solving the systems of linear algebraic equations (SLE). The dimensions of such systems can be so high that their solution becomes difficult or impossible without the use of special computing resources and tools [1, 4].

In turn, due to growing of need for powerful computational resources and lowering cost of distributed computing systems in comparison with specialized computing clusters, the relevance of the distributed computing systems has recently strongly increased [2].

Sectioning banded matrix – splitting it into such submatrices – sections, that fully include all nonzero elements.

The problem of sectioning high dimensional banded matrices appears, in the first place, if distributed or parallelized SLE solving is necessary, especially under conditions when the structure of matrix, which describes the system, is irregular, or unknown for some reasons [3].

Also need for matrices sectioning occurs in the implementation of iterative SLE solving, such as in paper [4] among others is mentioned splitting the matrix into sections when iteratively solving SLE on parallel systems with distributed memory, but details of the sectioning algorithm are not described.

Need for matrices sectioning occurs also in solving a number of specific tasks, such as designing computational processes anomalies detection systems, as mentioned in the paper [5], but no descriptions of specific algorithms are provided.

In the paper [6] considered the solution of the problem of finding the product of two real matrices on clusters of the serial personal computers, converted to a parallel system with shared memory (organized by the hard disk with network access), in particular, by using sectioning matrix multiplier.

Also solving of sectioned SLE by parallel system with shared memory is considered in the paper [7].

In the paper [8] mentioned sectioning matrix for parallel processing by the system with distributed memory and support of MPI standard.

However, in all the works matrices sectioning was not the main subject of research, but only one of the aspects, and therefore investigation of sectioning algorithms effectiveness was not presented.

The problem was stated and algorithms for high dimensional banded matrices sectioning in order to further distributed solving was developed for the first time in this work.

2. SECTIONING PROBLEM STATEMENT

Objective is to develop and compare algorithms for high dimensional banded matrices sectioning. The concept of sectioning and requirements to it should be considered in more detail.

The sectioning, or partitioning to sections, is said to be the finding for matrix A such natural vector $\vec{s} \in N^m$ of dimension m, which would simultaneously satisfy both conditions:

- sum of sizes of sections should equal the dimension of matrix A;

- any nonzero matrix A element must be inside the square area formed by two consecutive sections.

3. SECTIONING QUALITY CRITERIA

Since there may be many alternative sectionings of the matrix, it is important to compare them among themselves. The authors were invited to use these three criteria for evaluating the quality of matrix sectioning:

1) Sectioning smallness. Getting the most amount of small sections is very important to ensure flexibility of a SLE distributed solving, provide the most expedient tasks for performing systems and minimize total time losses in distributed system.
2) Sectioning accuracy. Since the computational complexity of full matrix of dimensions $n \times n$, which is, in the general case, each of the sections, diagonalization algorithms is $O(n^3)$, it is apparent that for maximum performance of distributed system sum of cubes of sizes of sections should be minimal.
3) Results precision. The results of solving the same SLE divided into the sections by several alternative methods may vary. If a reference SLE solution $\vec{x}$ is known, the precision of the results obtained by matrix sectioning $\vec{x}'$ can be defined as the mean square deviation (standard deviation) of vector $\vec{x}'$ from vector $\vec{x}$.

According to the proposed criteria for evaluating quality of matrix sectionings allow comparison of not only the alternative matrix sectionings, but comparison of the entire sectioning algorithms too.

4. SECTIONING ALGORITHMS

During the investigation of possible approaches to the high dimensional banded matrices sectioning the authors have developed three different algorithms, which are named according to the principle of action One-Directional (OD), Bidirectional (BD) and Adjustable (AD).

Each of three algorithms developed at each its execution step combines two sections into one. For each combining, all pairs of consecutive sections remaining at the current step are iterated through. Since the original sections count is n, then no more than $(n-1)$ combinings will be performed, for each of which, on average, $\frac{1}{2}(n-1)$ pairs of sections will be iterated through. Therefore, the computational complexity of algorithms developed is $O\left((n-1)\cdot \frac{1}{2}(n-1)\right) = O\left(\frac{1}{2}(n-1)^2\right) < O(n^2)$. And so we can say that all algorithms developed have the computational complexity that not exceeds $O(n^2)$.

It should also be noted that, if necessary, first step for each of these algorithms may be extended with matrix rows and columns ordering using the Cathill-McKee algorithm that will reduce the width of the matrix band and, hence, improve sectioning smallness and accuracy.

5. EXPERIMENTAL RESULTS

The testing of the developed algorithm was performed with six different SLE tests with sparse and banded matrices, such as SLE that arises in the real problems of thermal design of electronic devices. SLE tests consisted of 134, 1095, 1648, 3870, 6240 and 8192 equations.

During the test, sectioning for each of six test matrices was obtained using each of three developed sectioning algorithms. Then matrices were solved locally and distributedly, using obtained sectionings. For each of sectionings criteria *cnt*, *sqr* and *dev* was calculated and total time spent for SLE solving were determined.

All three matrices sectioning algorithms proposed confirmed their effectiveness. Since they all have computational complexity that not exceeds $O(n^2)$, it may be expedient to use several sectioning methods at a time and then select results that will be most convenient for this case. If it is necessary to choose a single sectioning algorithm, then Bidirectional (BD) seems to be most promising.

6. CONCLUSION

We have stated and proved relevance of the problem of high dimensional banded matrices sectioning when solving SLE distributedly.

The proposed sectioning quality criteria – sectioning smallness (criterion *cnt*), sectioning accuracy (criterion *sqr*) and results precision (criterion *dev*) – allow comparison of alternative matrix sectionings, just as comparison of entire sectioning algorithms.

One-Directional (OD), Bidirectional (BD) and Adjustable (AD) algorithms for matrices sectioning under research and development can perform partitioning matrices to sections, and has computing complexity that does not exceed $O(n^2)$.

The experimental results proved the effectiveness of all three developed matrices sectioning algorithms, especially of Bidirectional (BD).