MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

WEST UKRAINIAN NATIONAL UNIVERSITY

Methodical instructions for practical lessons in discipline «Higher Mathematics»

Ternopil – 2022

UDK 519.2

Reviewers:

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approved at the meeting of the Department of Applied Mathematics, protocol № 1 of 26.08.2022.

Methodical instructions for practical lessons in discipline «Higher Mathematics» include algorithms of solution different tasks in the discipline «*Higher Mathematics*» (for studing students to the practical lessons).

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Plaskon S.A., Dzyubanovska N.V. Methodical instructions for practical lessons in discipline «Higher Mathematics». - Ternopil: WUNU, 2022.- 17 p.

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The program and thematic plan is directed on deep and logical study of bases of higher mathematics and theory of chances, development of logical thought of students. This discipline behaves to general fundamental disciplines which form the world view of future economists and are basis of study of economical-mathematical design, and also economic disciplines (statistics, microeconomics, economic analysis and etc).

A main task the course of “Higher Mathematics” is a study of general conformities to the law and connection between the different sizes of their application to concrete economic researches. A capture a course must make for students skills of the practical use of mathematical methods, formulas and tables in the process of decision of economic tasks.

The purpose of course is forming of the system of theoretical knowledge and practical skills from bases of mathematical vehicle, basic methods of the quantitative measuring of chance of action of factors which influence on any processes, principles of mathematical statistics, which is used during planning, organization and management of operations, evaluation of quality of products, analysis of the systems of economic patterns and technological processes.

The study of course foresees the presence of systematic knowledge, purposeful prosecution of study of mathematical literature, active work on lectures and practical employments, independent work and processing of individual jobs.

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Instructional and methodical materials for conducting practical classes

Practical lesson 1.

Theme: Identifiers and their calculations - 2 hours.

Aim: To develop the skills of computing determinants of II, III and higher orders using the definition and their properties.

1. Definition of the second and third order, their calculation.

2. Schedule of determinants of III and higher orders for the elements of its tape (column).

Task 1. To calculate determinant of the second orde:

$$\begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix}.$$

Solution.

$$\begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix} = 3 \times 1 - (-4) \times 2 = 3 + 8 = 11.$$

Task 2. To calculate determinant of the third order

:

$$\begin{vmatrix} 2 & 1 & -3 \\ 3 & 0 & -1 \\ 4 & -2 & 5 \end{vmatrix}.$$

Solution.

$$\begin{vmatrix} 2 & 1 & -3 \\ 3 & 0 & -1 \\ 4 & -2 & 5 \end{vmatrix} = 2 \times 0 \times 5 + 1 \times (-1) \times 4 + (-3) \times 3 \times (-2) - (-3) \times 0 \times 4 - -1 \times 3 \times 5 - 2 \times (-1) \times (-2) = 0 - 4 + 18 + 0 - 15 - 4 = = -5.$$

Task 3. To calculate determinant of the third order, decomposing him after the elements of line (or column):

$$\begin{vmatrix} 1 & 2 & 4 \\ 3 & -1 & 0 \\ 1 & 2 & -5 \end{vmatrix}.$$

Solution.

$$\begin{vmatrix} 1 & 2 & 4 \\ 3 & -1 & 0 \\ 1 & 2 & -5 \end{vmatrix} = 3\cdot(-1)^{2+1}\cdot\begin{vmatrix} 2 & 4 \\ 2 & -5 \end{vmatrix} + (-1)\cdot(-1)^{2+2}\cdot\begin{vmatrix} 1 & 4 \\ 1 & -5 \end{vmatrix} +$$

$$+ 0\cdot(-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 3\cdot(-1)^{3}\cdot(-10-8) - 1\cdot(-1)^{4}\cdot(-5-4) + 0 =$$

$$= -3\cdot(-18) - 1\cdot(-9) = 63.$$

Task 4. To calculate determinant of fourth order, using him to property:

$$\begin{vmatrix} 3 & 2 & 1 & 4 \\ 1 & 0 & 1 & 2 \\ -1 & -1 & 3 & 0 \\ 0 & 2 & 1 & 5 \end{vmatrix}.$$

Solution.

$$\begin{vmatrix} 3 & 2 & 1 & 4 \\ 1 & 0 & 1 & 2 \\ -1 & -1 & 3 & 0 \\ 0 & 2 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 0 & 2 & -2 & -2 \\ 1 & 0 & 1 & 2 \\ 0 & -1 & 4 & 2 \\ 0 & 2 & 1 & -5 \end{vmatrix} = (-1)^{3}\begin{vmatrix} 2 & -2 & -2 \\ -1 & 4 & 2 \\ 2 & 1 & -5 \end{vmatrix} =$$

$$= -\begin{vmatrix} 2 & 0 & 0 \\ -1 & 3 & 1 \\ 2 & 3 & -3 \end{vmatrix} = -2\cdot(-1)^{2}\cdot\begin{vmatrix} 3 & 1 \\ 3 & -3 \end{vmatrix} = -2\cdot(-9-3) = 24.$$

Practical lesson 2.

Topic: Matrix and action on them - 2 hours.

Objective: To teach to perform actions on matrices (addition, subtraction, multiplication by number, multiplication of matrices, finding the inverse matrix, rank finding). Develop the ability to apply the matrix when solving economic problems.

1. Actions on matrices.

2. An inverse matrix and its location.

3. Rank of the matrix and its location.

4. Economic problems using the theory of matrices.

Task 1. Find a product $A \cdot B$ if

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}.$$

Solution.

$$A \cdot B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix} =$$

$$= \begin{bmatrix} 1\cdot(-2)+2\cdot 3+3\cdot 1 & 1\cdot 1+2\cdot 2+3\cdot 3 & 1\cdot 2+2\cdot 1+3\cdot 2 \\ 4\cdot(-2)+5\cdot 3+6\cdot 1 & 4\cdot 1+5\cdot 2+6\cdot 3 & 4\cdot 2+5\cdot 1+6\cdot 2 \\ 2\cdot(-2)+1\cdot 3+4\cdot 1 & 2\cdot 1+1\cdot 2+4\cdot 3 & 2\cdot 2+1\cdot 1+4\cdot 2 \end{bmatrix} =$$

$$= \begin{bmatrix} 7 & 14 & 10 \\ 13 & 32 & 25 \\ 3 & 16 & 13 \end{bmatrix}.$$

Task 2. Find a matrix rank

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 6 & -3 & -1 \\ 3 & 6 & -3 & 10 \end{bmatrix}.$$

Solving The rank of the matrix will be searched by the elementary transformation method.

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 6 & -3 & -1 \\ 3 & 6 & -3 & 10 \end{bmatrix} \overset{(-3)}{\curvearrowright} \Leftrightarrow \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 0 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Leftrightarrow$$

$$\Leftrightarrow\begin{bmatrix}1 & 2 & -1 & 3\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & -10\end{bmatrix} \quad 10 \qquad \Leftrightarrow\begin{bmatrix}1 & 2 & -1 & 3\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}\Leftrightarrow$$

$$\Leftrightarrow\begin{bmatrix}1 & 3 & -1 & 2\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}.$$

Hence it follows that the rank of this matrix is 2 (below the main diagonal □ zeros and two elements of the main diagonal), $\text{rang}(A)=2$

Task 3. Find an inverse matrix to a matrix

$$A=\begin{bmatrix}-2 & 3 & 4\\ 3 & -1 & -3\\ -1 & 2 & 2\end{bmatrix}.$$

Solving. First, make sure the matrix has a reverse A^{-1}. Identifier

$$|A|=\begin{vmatrix}-2 & 3 & 4\\ 3 & -1 & -3\\ -1 & 2 & 2\end{vmatrix}=4+24+9-4-18-12=3\neq 0.$$

So, the matrix has an inverse. We find algebraic additions to the elements of the matrix:

$$A_{11}=(-1)^2\begin{vmatrix}-1 & -3\\ 2 & 2\end{vmatrix}=-2-(-6)=4;$$

$$A_{12}=(-1)^3\begin{vmatrix}3 & -3\\ -1 & 2\end{vmatrix}=-(6-3)=-3;$$

$$A_{13}=(-1)^4\begin{vmatrix}3 & -1\\ -1 & 2\end{vmatrix}=6-1=5;$$

$$A_{21}=-\begin{vmatrix}3 & 4\\ 2 & 2\end{vmatrix}=2; \qquad A_{22}=\begin{vmatrix}-2 & 4\\ -1 & 2\end{vmatrix}=0;$$

$$A_{23} = -\begin{vmatrix} -2 & 3 \\ -1 & 2 \end{vmatrix} = 1; \qquad A_{31} = \begin{vmatrix} 3 & 4 \\ -1 & -3 \end{vmatrix} = -5;$$

$$A_{32} = -\begin{vmatrix} -2 & 4 \\ 3 & -3 \end{vmatrix} = 6; \qquad A_{33} = \begin{vmatrix} -2 & 3 \\ 3 & -1 \end{vmatrix} = -7.$$

The matrix of algebraic additions will be

$$\overline{A} = \begin{bmatrix} 4 & -3 & 5 \\ 2 & 0 & 1 \\ -5 & 6 & -7 \end{bmatrix}.$$

The attached matrix has the form:

$$A^* = \begin{bmatrix} 4 & 2 & -5 \\ -3 & 0 & 6 \\ 5 & 1 & -7 \end{bmatrix}.$$

So, we get

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 4 & 2 & -5 \\ -3 & 0 & 6 \\ 5 & 1 & -7 \end{bmatrix}.$$

Practical lesson 3.

Topic: Matrix analysis in economics - 2 hours.

Objective: To teach to solve systems of linear algebraic equations by the methods of Kramer, Gauss, Jordan-Gauss, using an inverse matrix. To get acquainted with the matrix models of the economy: Leontyev's model of inter-industry balance, finding of raw materials, fuel and labor resources and methods of their solution.

1. The concept of systems of linear algebraic equations.
2. The Cramer's Rule.
3. Gauss and Jordan-Gauss method.
4. Matrix method for solving equations.
5. Matrix model of Leontiev interbranch balance.
6. The task of finding the costs of raw materials, fuel and labor resources.

Task 1. To solve the system of equations according to the Cramer's rule

$$\begin{cases} x_1 + 2x_2 - x_3 = -3 \\ 2x_1 + 3x_2 + x_3 = -1 \\ x_1 - x_2 - x_3 = 3 \end{cases}.$$

Solution.

$$\Delta = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{vmatrix} = -3 + 2 + 2 + 3 + 4 + 1 = 9 \neq 0.$$

$$x_1 = \frac{\Delta_1}{\Delta},\ x_2 = \frac{\Delta_2}{\Delta},\ x_3 = \frac{\Delta_3}{\Delta}.$$

$$\Delta_1 = \begin{vmatrix} -3 & 2 & -1 \\ -1 & 3 & 1 \\ 3 & -1 & -1 \end{vmatrix} = 9 - 1 + 6 + 9 - 2 - 3 = 18,$$

$$\Delta_2 = \begin{vmatrix} 1 & -3 & -1 \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = 1 - 6 - 3 - 1 - 6 - 3 = -18,$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{vmatrix} = 9 + 6 - 2 + 9 - 12 - 1 = 9.$$

$$x_1 = \frac{18}{9} = 2;\quad x_2 = \frac{-18}{9} = -2;\quad x_3 = \frac{9}{9} = 1.$$

So, the solution of the given system will be (2; –2; 1).

Task **2.** Solve a matrix system with a system of equations

$$\begin{cases} 2x_1 + 3x_2 + 2x_3 = 9 \\ x_1 + 2x_2 - 3x_3 = 14 \\ 3x_1 + 4x_2 + x_3 = 16 \end{cases}.$$

Solution.

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 2 & -3 \\ 3 & 4 & 1 \end{bmatrix}.$$

$$|A| = \begin{vmatrix} 2 & 3 & 2 \\ 1 & 2 & -3 \\ 3 & 4 & 1 \end{vmatrix} = 4 + 8 - 27 - 12 - 3 + 24 = -6 \neq 0.$$

$A_{11} = \begin{vmatrix} 2 & -3 \\ 4 & 1 \end{vmatrix} = 14$; $\qquad A_{12} = -\begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix} = -10;$

$A_{13} = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2$; $\qquad A_{21} = 5$; $\qquad A_{22} = -4$;

$A_{23} = 1$; $\qquad A_{31} = -13$; $\qquad A_{32} = 8$; $\qquad A_{33} = 1$.

$$\overline{A} = \begin{bmatrix} 14 & -10 & -2 \\ 5 & -4 & 1 \\ -13 & 8 & 1 \end{bmatrix}.$$

$$A^* = \begin{bmatrix} 14 & 5 & -13 \\ -10 & -4 & 8 \\ -2 & 1 & 1 \end{bmatrix}.$$

$$A^{-1} = -\frac{1}{6}\begin{bmatrix} 14 & 5 & -13 \\ -10 & -4 & 8 \\ -2 & 1 & 1 \end{bmatrix}.$$

$$X = -\frac{1}{6}\cdot\begin{bmatrix} 14 & 5 & -13 \\ -10 & -4 & 8 \\ -2 & 1 & 1 \end{bmatrix}\cdot\begin{bmatrix} 9 \\ 14 \\ 16 \end{bmatrix} = -\frac{1}{6}\cdot\begin{bmatrix} 14\cdot 9+5\cdot 14+(-13)\cdot 16 \\ (-10)\cdot 9+(-4)\cdot 14+8\cdot 16 \\ (-2)\cdot 9+1\cdot 14+1\cdot 16 \end{bmatrix} = = -\frac{1}{6}\cdot\begin{bmatrix} -12 \\ -18 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}.$$

The solution of the system will be: $x_1 = 2,\ x_2 = 3,\ x_3 = -2.$

∢ Ex. 1 **In a rhombus $ABCD$ diagonals are set $\overrightarrow{AC} = \vec{a}$ and $\overrightarrow{BD} = \vec{b}$. To decompose after these two vectors all of vectors which coincide with the sides of rhombus: $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$.**

Instruction.

We consider a rhombus $ABCD$.

$\overrightarrow{AO} = \frac{1}{2}\overrightarrow{AC} = \frac{1}{2}\vec{a};$

$\overrightarrow{BO} = \frac{1}{2}\overrightarrow{BD} = \frac{1}{2}\vec{b};$

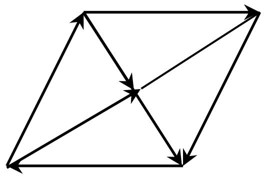
$\overrightarrow{AO} = \overrightarrow{AB} + \overrightarrow{BO};$

$\overrightarrow{AB} = \overrightarrow{AO} - \overrightarrow{BO} = \frac{1}{2}(\vec{a} - \vec{b});$

$\overrightarrow{CD} = -\overrightarrow{AB} = -\frac{1}{2}(\vec{a} - \vec{b});$

$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} = \vec{a} - \frac{1}{2}(\vec{a} - \vec{b}) = \frac{1}{2}(\vec{a} + \vec{b});$

$\overrightarrow{DA} = -\overrightarrow{BC} = -\frac{1}{2}(\vec{a} + \vec{b}).$



∢ Task 2. **A vector** $\vec{a}$ **is set the coordinates of the ends** $A(1; 3; -2)$ **and** $B(2; -1; 5)$. **Please, define coordinates, length and direction of this vector.**

Instruction. Find the coordinates of vector $\vec{a}$ **as difference between the eventual and initial coordinates of points:** $X = 2 - 1 = 1$**;** $Y = -1 - 3 = -4$**;** $Z = 5 - (-2) = 7$**.**

Length of vector.

$$|\vec{a}| = \sqrt{X^2 + Y^2 + Z^2} = \sqrt{1^2 + (-4)^2 + 7^2} = \sqrt{1 + 16 + 49} = \sqrt{66}\,.$$

Direction of vector is determined by sending cosines:

$$\cos\alpha = \frac{X}{|a|} = \frac{1}{\sqrt{66}}\,;\ \ \cos\beta = \frac{-4}{\sqrt{66}}\,;\ \ \cos\gamma = \frac{7}{\sqrt{66}}\,.$$

For verification of our results we'll find:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$$

$$= (\frac{1}{\sqrt{66}})^2 + (-\frac{4}{\sqrt{66}})^2 + (\frac{7}{\sqrt{66}})^2 = \frac{1 + 16 + 49}{66} = 1.$$

* *

∢ **Practice 1.** Plane goes through point $P(3; 6; -4)$ and separate segments on the axis of absciss $a = -3$ and on the z-axis $c = 2$. Write equation of the plane.

Aanswer*:* We have to use $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. If $a = -3$, $c = 2$, then $\frac{x}{-3} + \frac{y}{b} + \frac{z}{2} = 1$.

Point P is on the plane, that is why it coordinates satisfy equation of this plane:

$$\frac{3}{-3} + \frac{6}{b} + \frac{-4}{2} = 1, \text{ so } b = \frac{3}{2}.$$

Equation of the plane will be $\frac{x}{-3} + \frac{2y}{3} + \frac{z}{2} = 1$, or

$$2x - 4y - 3z + 6 = 0.$$

∢ **Practice 2.** Find distance between point $A(2;3;-1)$ and a plane $7x-6y-6z+42=0$.

Answer. We use this formula for finding distance between point and plane

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}.$$

Having substituted into the formula values $A=7;\ B=-6;\ C=-6;\ x_0=2;\ y_0=3;\ z_0=-1$, receive

$$d=\left|\frac{7\cdot 2+(-6)\cdot 3+(-6)(-1)+42}{\sqrt{7^2+(-6)^2+(-6)^2}}\right|=\left|\frac{14-18+6+42}{11}\right|=4.$$

* *
*

∢ **Task 1.**To define, what values x inequality is executed at $|x-3|<2$.

Soluting. The set inequality can be written down so: $-2<x-3<2$. To every part of this inequality will add for 3 and obsessed $-2+3<x<2+3$, that $1<x<5$. Consequently, inequality $|x-3|<2$ executed for all values x from an interval $(1,5)$.

∢ **Task 2.**To find the range of definition of function $y=\sqrt{2-x}$.

*Untiing.*In order that a function y had actual values only, size $2-x$, that is under a root, must not have subzero values, but must be $2-x\ge 0$, that $x\le 2$. The range of definition of function is an aggregate of actual values x that less or evened 2, that $x\in(-\infty;2]$.

∢ **Task 1.** To find the derivative of function $y=3x^2$ at $x=4$.

Untiing. Will find the decision of this task, going out from determination. If argument x gets an increase Δx, то для функції $y=f(x)=3x^2$ will find an increase Δy, that

$$f(x+\Delta x)=3(x+\Delta x)^2=3x^2+6x\Delta x+3(\Delta x)^2,$$
$$\Delta y=f(x+\Delta x)-f(x)=3x^2+6x\Delta x+3(\Delta x)^2-3x^2=$$
$$=6x\Delta x+3(\Delta x)^2=(6x+3\Delta x)\Delta x.$$

Will divide the increase of function Δy on the increase of argument Δx, that will find middle speed of change of the set function $y=3x^2$ on an interval $(x,x+\Delta x)$.

For finding of derivative y' it is needed to find granicyu of the got relation at $\Delta x\to 0$ (here x it is considered a permanent size). Thus

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{(6x + 3\Delta x)\Delta x}{\Delta x} = \lim_{\Delta x \to 0} (6x + 3\Delta x) = 6x.$$

At $x = 4$ value of derivative $y'(4) = 6 \cdot 4 = 24$. It a number 24 is speed of change of function $y = 3x^2$ at $x = 4$.

∢ **Task 2.** **To find the derivatives of functions**:

а) $y = \sqrt{x^2 + 1} + \sqrt[3]{x^3 + 1}$; б) $y = \ln\sqrt{\dfrac{1 - \sin x}{1 + \sin x}}$.

Untiing.

a) Will use the rule of differentiation for the sum of two differentiated functions, and then will find the derivatives of difficult functions:

$$y' = (\sqrt{x^2 + 1} + \sqrt[3]{x^3 + 1})' = (\sqrt{x^2 + 1})' + (\sqrt[3]{x^3 + 1})' = \left((x^2 + 1)^{\frac{1}{2}}\right)' +$$

$$+\left((x^3 + 1)^{\frac{1}{3}}\right)' = \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}}(x^2 + 1)' + \frac{1}{3}(x^3 + 1)^{-\frac{2}{3}}(x^3 + 1)' =$$

$$= \frac{1}{2\sqrt{x^2 + 1}} \cdot 2x + \frac{1}{3\sqrt[3]{(x^3 + 1)^2}} \cdot 3x^2 = \frac{x}{\sqrt{x^2 + 1}} + \frac{x^2}{\sqrt[3]{(x^3 + 1)^2}}.$$

b) **Prologarifmuemo** the set function, and then will find the derivative of difficult function:

$$y = \ln\sqrt{\frac{1 - \sin x}{1 + \sin x}} = \ln\left(\frac{1 - \sin x}{1 + \sin x}\right)^{\frac{1}{2}} = \frac{1}{2}\ln(1 - \sin x) - \frac{1}{2}\ln(1 + \sin x);$$

$$y' = \left(\frac{1}{2}\ln(1 - \sin x) - \frac{1}{2}\ln(1 + \sin x)\right)' = \left(\frac{1}{2}\ln(1 - \sin x)\right)' -$$

$$-\left(\frac{1}{2}\ln(1 + \sin x)\right)' = \frac{1}{2}\frac{(1 - \sin x)'}{1 - \sin x} - \frac{1}{2}\frac{(1 + \sin x)'}{1 + \sin x} = \frac{1}{2}\frac{-\cos x}{1 - \sin x} -$$

$$-\frac{1}{2}\frac{\cos x}{1 + \sin x} = \frac{-2\cos x}{2(1 - \sin^2 x)} = -\frac{1}{\cos x}.$$

∢ **Task 3.** To find the derivative of the third order of function $y = \sin^2 x$.

Untiing. *Will find the derivative of the first order, as a derivative of function of degree:*

$$y' = (\sin^2 x)' = \left[(\sin x)^2\right]' = 2\sin x(\sin x)' = 2\sin x \cdot \cos x = \sin 2x.$$

Find the derivative of the second order as a derivative from the found result for y', that $y'' = (y')'$. like $y''' = (y'')'$. Consequently,

$$y'' = (\sin 2x)' = \cos 2x(2x)' = 2\cos 2x;$$
$$y''' = (2\cos 2x)' = 2(-\sin 2x)(2x)' = -4\sin 2x.$$

∢ Task 4. To find a derivative y' non-obvious function

$$x^2 + y^2 = 9.$$

Untiing. In the set equalization a function is found ambiguously, that is why it is named non-obvious. Differentiating both parts of equality, obsessed $2x + 2yy' = 0.$ From here have $yy' = -x.$ Deciding this equalization relatively y', find, that $y' = -\frac{x}{y}$.

Here at differentiation of second addition $\left[(y^2)'_x = 2yy'\right]$ the derivative of function of degree is at first found, and basis is later differentiated y пo to the independent variable x (that y').

Task 1. To find an indefinite integral

$$\int (4x^3 + \frac{1}{2\sqrt{x}} - 5\sqrt[3]{x^2})dx.$$

Untiing. Using formulas (1), (16) i (17) tables, obsessed:

$$\int (4x^3 + \frac{1}{2\sqrt{x}} - 5\sqrt[3]{x^2})dx = \int 4x^3 dx + \int \frac{1}{2\sqrt{x}} dx - \int 5\sqrt[3]{x^2}\, dx = 4\int x^3 dx +$$

$$+\frac{1}{2}\int x^{-\frac{1}{2}} dx - 5\int x^{\frac{2}{3}} dx = 4\cdot\frac{x^4}{4} + \frac{1}{2}\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 5\cdot\frac{x^{\frac{5}{3}}}{\frac{5}{3}} + C = x^4 + \sqrt{x} - 3x\sqrt[3]{x^2} + C.$$

∢ Task 2. To find an indefinite integral $\int \frac{xdx}{x^2 - 5}$.

Solution. As $xdx = \frac{1}{2} d(x^2 - 5),$ bringing an integral over to to tabular, have

$$\int \frac{xdx}{x^2 - 5} = \frac{1}{2}\int \frac{d(x^2 - 5)}{x^2 - 5} = = \frac{1}{2}\ln\left|x^2 - 5\right| + C.$$

Task 1. To find an indefinite integral

$$\int (4x^3 + \frac{1}{2\sqrt{x}} - 5\sqrt[3]{x^2})dx.$$

Untiing. Using formulas (1), (16) i (17) tables, obsessed:

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$$+\frac{1}{2}\int x^{-\frac{1}{2}}dx-5\int x^{\frac{2}{3}}dx=4\cdot\frac{x^4}{4}+\frac{1}{2}\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}}-5\cdot\frac{x^{\frac{5}{3}}}{\frac{5}{3}}+C=x^4+\sqrt{x}-3x\sqrt[3]{x^2}+C.$$

∢ **Task 2. To find an indefinite integral** $\int\frac{xdx}{x^2-5}$.

Untiing. As $xdx=\frac{1}{2}d(x^2-5)$, bringing an integral over to to tabular, have

$$\int\frac{xdx}{x^2-5}=\frac{1}{2}\int\frac{d(x^2-5)}{x^2-5}==\frac{1}{2}\ln\left|x^2-5\right|+C.$$

Task 1. To calculate a certain integral $\int_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}}\frac{4dx}{\sqrt{1-x^2}}$.

Untiing. Using property 2.5, will find primitive to the function:

$$\int_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}}\frac{4dx}{\sqrt{1-x^2}}=4\int_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}}\frac{dx}{\sqrt{1-x^2}}=4\arcsin x\Big|_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}}=4(\arcsin\frac{\sqrt{2}}{2}-$$

$$-\arcsin\frac{1}{2})=4\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\frac{\pi}{3}.$$

Task 2. To calculate a certain integral

$$\int_0^{\frac{\pi}{2}}\cos^3 x\sin xdx.$$

Untiing. Will do replacement of variable, will put $t=\cos x.$ Then $dt=-\sin xdx,$ and $\sin xdx=-dt.$ will Find the new limits of integration:

If $x=0$, then $t=\cos 0=1;$

If $x=\frac{\pi}{2}$, then $t=\cos\frac{\pi}{2}=0$

By such rank

$$\int_0^{\frac{\pi}{2}} \cos^3 x \sin x dx = -\int_1^0 t^3 dt = -\frac{1}{4} t^4 \Big|_1^0 = -\frac{1}{4}(0^4 - 1^4) = \frac{1}{4}.$$

Task 3. To calculate a certain integral $\int_0^1 xe^{-x} dx.$

Untiing. Will use the method of integration parts. Will put $u = x,\ dv = e^{-x} dx$, then

$$du = dx,\ v = \int e^{-x} dx = -\int e^{-x} dx = -e^{-x}.$$