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FREQUENCY NOISE INFLUENCE ON FUNCTIONING OF FREDKIN QUANTUM GATE

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Abstract: *The influence of detuning of radio frequency magnetic field (RFMF) on the functioning of nuclear magnetic resonance (NMR) quantum Fredkin gate is considered in this paper. It is shown that detuning of frequency decreases a probability of correct answer. If the spectral broadband of RFMF signal is increasing, then the main value of correct answer probability is decreasing too and standard deviation of this probability is increasing.*

Keywords: *quantum bit, quantum register, quantum computer, quantum algorithm, spin, Ising model, Gaussian frequency band, correct answer probability, probability standard deviation.*

The hopes of scientist and experts to quantum computer (QC) lay in quantum parallelism phenomenon [1,2]. Quantum bit (QB) in contrast to classic one may be simultaneously in two states: logical 0 and logical 1 with equal probabilities. Quantum n-tuple register may keep and process 2^n binary words simultaneously. In this sense every quantum algorithm (QA) is the sequence of allowed quantum transitions between probable quantum register Boolean states. Now two QAs are developed: Grover's database search algorithm and Shore's big integer numbers factorization algorithm [3].

QB may be represented by every quantum object with two quantum states, which are sharply separated in energies. If QBs are interacting electron or nuclear spins in strong constant magnetic field (CMF) and RFMF, that is QC on NMR.

The aim of this paper is investigation of RFMF detuning influence on Fredkin quantum gate functioning in chain of three nuclear or electron spins ½ in Ising model framework. Fredkin gate swaps two controlled bits if controlling bit is in logical 1 state and does not swapping in opposite case.

Evolution of three spins system has been investigated by the model Hamiltonian [4]:

$$H = H_0 + W\,, \tag{1}$$

where:

$$H_0 = -\hbar\left\{\sum_{k=0}^{2}\omega_k I_k^z + 2J\left[I_0^z I_1^z + I_1^z I_2^z + \alpha I_0^z I_2^z\right]\right\}, \tag{2}$$

$$W = -\frac{\hbar\Omega}{2}\sum_{k=0}^{2}\left[I_k^+ \exp(i\omega t) + I_k^- \exp(-i\omega t)\right]. \tag{3}$$

The Hamiltonian describes the behavior of three interacting spins system in magnetic field $B = (b\cos\omega t, -b\sin\omega t, B(z))$, where b and ω are amplitude and frequency of RFMF respectively, $B(z)$ – z-coordinate dependent CMF induction, I_k^z – projection of k-th spin on z-axis, $\omega_k = \gamma B(z_k)$ – Larmore's precession frequencies for corresponding spins, γ – proton or electron gyromagnetic ratio, J – exchange interaction constant for nearest neighbors, α – relative exchange interaction for second neighbors, $I_k^{\pm}$ – so called "descend" and "ascend" operators, $\Omega = \gamma b$ – Rabi's frequency, $\hbar$ – Plank's constant, divided by 2π.

Analyzed system has eight basic states, which may be numbered by binary numbers from 0 to 7. Evolution of the system may be described by time-dependent Schrödinger equation (TDSE), which may be represented in such matrix form:

$$\dot{D} = T(t)D\,, \tag{4}$$

where D – unknown time-dependent matrix of basic functions expansion coefficients for system wave function. The time-dependent matrix $T(t)$ has the form:

$$T_{mn}(t) = -\frac{i}{\hbar} W_{mn}(t) \exp(i\omega_{mn} t), \quad (5)$$

where matrix $W(t)$ has the form:

$$W(t) = -\frac{\hbar\Omega}{2} \begin{pmatrix} 0 & z^* & z^* & 0 & z^* & 0 & 0 & 0 \\ z & 0 & 0 & z^* & 0 & z^* & 0 & 0 \\ z & 0 & 0 & z^* & 0 & 0 & z^* & 0 \\ 0 & z & z & 0 & 0 & 0 & 0 & z^* \\ z & 0 & 0 & 0 & 0 & z^* & z^* & 0 \\ 0 & z & 0 & 0 & z & 0 & 0 & z^* \\ 0 & 0 & z & 0 & z & 0 & 0 & z^* \\ 0 & 0 & 0 & z & 0 & z & z & 0 \end{pmatrix}, \quad (6)$$

$z = \exp(i\omega t)$ and asterisk defines the complex conjugation. Numbers ω_{mn} may be defined as:

$$\omega_{mn} = \frac{1}{\hbar}(E_m - E_n). \quad (7)$$

If $(i_2 i_1 i_0)$ is binary representation of number $m(m = 0...7)$, then corresponding eigenvalues of energy E_m may be defined as:

$$E_{i_2 i_1 i_0} = -\frac{\hbar}{2}\Big\{(-1)^{i_2}\omega_2 + (-1)^{i_1}\omega_1 + (-1)^{i_0}\omega_0 + J\Big[(-1)^{i_0+i_1} + (-1)^{i_1+i_2} + \alpha(-1)^{i_0+i_2}\Big]\Big\}. \quad (8)$$

Formal solution of (4) may be represented as:

$$D(t) = D(0)\exp\left(\int_0^t T(t)dt\right). \quad (9)$$

Time-depended probabilities of realization for every probable state m of the system may be defined as $|D_m(t)|^2$.

If we want to analyze the transition between L and M states we must put in to $T(t)$ “resonant” RFMF frequency value $\omega = |\omega_{LM}|$. For Fredkin gate numerical simulation we considered the transition sequences 5→7→6 and 5→7→3 because only quantum transitions with inversion of one bit are allowed. System parameters are (in units 2π MHz): $\omega_0 = 100, \omega_1 = 200, \omega_2 = 400, J = 5, \alpha = 0.02$, $\Omega = 0.1$. It was shown, that Fredkin gate without detuning acts sharply during two π-pulses, which duration is π/Ω, i.e. probability of final state realization becomes equal to 1. For detuning analysis we considered three detuning mechanisms: i) RFMF frequency tuning error, when $\omega = |\omega_{LM}|(1+\eta_1)$, where η_1 is relative detuning; ii) non-controlled pure shift of system energy levels Δ_i due to interaction of the system with environment, when $\Delta_0 = 0, \Delta_k = \eta_2(E_k - E_{k+1})$ for $k = 1...7$, where η_2 is relative shift; iii) finite broad of signal spectrum, when $\omega = |\omega_{LM}| Ga(1,\eta_3)$, where $Ga(1,\eta_3)$– normally distributed random numbers with centre 1 and standard deviation η_3. In case i) the correct answer probability becomes equal to 0.5 if $|\eta_1| \approx 6.2\cdot 10^{-4}$ for sequence 5→7→6 and if $|\eta_2| \approx 3\cdot 10^{-4}$ for sequence 5→7→3. Such difference occurs because the energy interval 5→6 is less then 5→3 one. In this case the correct answer probability for both sequences has not only leading maximum at $\eta_1 = 0$, but also side maximums. In case ii) the correct answer probability becomes equal to 0.5 if $|\eta_2| \approx 0.012$ for both sequences. For this case correct answer probability has side maximums too. In case iii) the main probability of correct answer becomes equal to its standard deviation at $\eta_3 \approx 5\cdot 10^{-4}$ for sequence 5→7→6 and at $\eta_3 \approx 1.6\cdot 10^{-3}$ for sequence 5→7→3. In last case we considered the ensemble of 20 realizations of Fredkin quantum gate action.

We can conclude that probability of correct answer for Fredkin quantum gate is very sensitive to RFMF frequency detuning. Therefore for complicated algorithms the interim results correction procedures are required.

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